Exercise 1.53

Given two vectors $\vec{\boldsymbol{A}} = -2.00\hat{\boldsymbol{i}} + 3.00\hat{\boldsymbol{j}} + 4.00\hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{B}} = 3.00\hat{\boldsymbol{i}} + 1.00\hat{\boldsymbol{j}} - 3.00\hat{\boldsymbol{k}}$, do the following. (a) Find the magnitude of each vector. (b) Write an expression for the vector difference $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$ using unit vectors. (c) Find the magnitude of the vector difference $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$. Is this the same as the magnitude of $\vec{\boldsymbol{B}} - \vec{\boldsymbol{A}}$? Explain.

Solution

The magnitudes of the given vectors are

$$\begin{aligned}|\mathbf{A}| &= \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = \sqrt{29} \approx 5.39 \\ |\mathbf{B}| &= \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = \sqrt{19} \approx 4.36,\end{aligned}$$

the vector difference $\mathbf{A} - \mathbf{B}$ is

$$\begin{aligned}\mathbf{A} - \mathbf{B} &= (-2.00\hat{\boldsymbol{i}} + 3.00\hat{\boldsymbol{j}} + 4.00\hat{\boldsymbol{k}}) - (3.00\hat{\boldsymbol{i}} + 1.00\hat{\boldsymbol{j}} - 3.00\hat{\boldsymbol{k}}) \\ &= (-2.00 - 3.00)\hat{\boldsymbol{i}} + (3.00 - 1.00)\hat{\boldsymbol{j}} + [4.00 - (-3.00)]\hat{\boldsymbol{k}} \\ &= -5.00\hat{\boldsymbol{i}} + 2.00\hat{\boldsymbol{j}} + 7.00\hat{\boldsymbol{k}},\end{aligned}$$

and its magnitude is

$$\begin{aligned}|\mathbf{A} - \mathbf{B}| &= \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} \\ &= \sqrt{78} \\ &\approx 8.83.\end{aligned}$$

On the other hand, the vector difference $\mathbf{B} - \mathbf{A}$ is

$$\begin{aligned}\mathbf{B} - \mathbf{A} &= (3.00\hat{\boldsymbol{i}} + 1.00\hat{\boldsymbol{j}} - 3.00\hat{\boldsymbol{k}}) - (-2.00\hat{\boldsymbol{i}} + 3.00\hat{\boldsymbol{j}} + 4.00\hat{\boldsymbol{k}}) \\ &= [3.00 - (-2.00)]\hat{\boldsymbol{i}} + (1.00 - 3.00)\hat{\boldsymbol{j}} + (-3.00 - 4.00)\hat{\boldsymbol{k}} \\ &= 5.00\hat{\boldsymbol{i}} - 2.00\hat{\boldsymbol{j}} - 7.00\hat{\boldsymbol{k}},\end{aligned}$$

and its magnitude is

$$\begin{aligned}|\mathbf{B} - \mathbf{A}| &= \sqrt{(5.00)^2 + (-2.00)^2 + (-7.00)^2} \\ &= \sqrt{78} \\ &\approx 8.83.\end{aligned}$$

The magnitudes of $\mathbf{B} - \mathbf{A}$ and $\mathbf{A} - \mathbf{B}$ are the same.